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Topological defects with long-range interactions

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Abstract

We investigate a modified sine-Gordon equation which possesses soliton solutions with long-range interaction. We introduce a generalized version of the Ginzburg–Landau equation which supports long-range topological defects in $D = 1$ and $D > 1$. The interaction force between the defects decays so slowly that it is possible to enter the non-extensivity regime. These results can be applied to non-equilibrium systems, pattern formation and growth models. © 1998 Elsevier Science B.V.

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1. Introduction

In recent years there has been a great interest in the interaction of topological defects [1–9]. In particular, it is very important to formulate models in which the solitons can interact with long-range forces [6]. This is due to several reasons. The majority of the known models supports solitons that interact with short-range forces [6]. However, the real transfer mechanisms are long-range [9–11]. This can be observed in a great number of physical systems, including condensed matter theory [11], spin glasses [13], neural systems [14], biological systems [10,11,15–17], DNA dynamics [7,15–17], etc.

On the other hand, it is very relevant per se to have models where there is spontaneous formation of particle-like structures that possess long-range interaction. These models would allow one to study pattern formation and other complex phenomena. It is well-known that systems with long-range microscopic interactions can exhibit non-extensive behavior [18,19]. For these systems new statistical theories have been proposed [20] and they require verification.

Recently, some authors have considered long-range effects [9] using ad hoc nonlocal terms in the equations. Spin systems have also been studied, where the coupling constant J_{ij} between the lattice spins is a rational function of coordinates [12].

In Ref. [6] González and Estrada-Sarlabous showed for the first time that pure Klein–Gordon systems,

$$\phi_{tt} - \phi_{xx} = G(\phi), \tag{1}$$

where $G(\phi) = -\partial U(\phi)/\partial\phi$, without coordinate-dependent terms, can support solitons with long-range interactions. In Eq. (1) we assume that potential

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$U(\phi)$ possesses at least two minima (in points ϕ_1 and ϕ_3), in a neighborhood of which

$$U(\phi) \sim (\phi - \phi_i)^{2n}. \tag{2}$$

For $n = 1$ the solitons interact with short-range forces (which decay exponentially). For $n > 1$, the solitons interact with forces that decay with the distance d as

$$F \sim d^{2n/(1-n)}. \tag{3}$$

In the present Letter we investigate a system of type (1) that is a generalization of the sine-Gordon equation. We will show that it possesses solitonic solutions that present long-range interaction.

As an example of systems with dimension $D > 1$, which supports topological defects whose interaction force decays very slowly we introduce a generalized version of the Ginzburg–Landau equation. We are able to create a gas of topological defects with an interaction force that decays so slowly that we enter the interesting regime of non-extensivity.

Finally, we will show how these results can be applied to non-equilibrium systems, pattern formation and growth models and some specific physical systems.

2. Klein–Gordon equation

In this section we will study the equation [8]

$$\phi_{tt} - \phi_{xx} + 2\sin^{2n-1}\left(\tfrac{1}{2}\phi\right)\cos\left(\tfrac{1}{2}\phi\right) = 0. \tag{4}$$

Eq. (4) can be derived from the Lagrangian density

$$L = \tfrac{1}{2}\phi_t^2 - \tfrac{1}{2}\phi_x^2 - U(\phi), \tag{5}$$

where

$$U(\phi) = \frac{2}{n}\sin^{2n}\left(\tfrac{1}{2}\phi\right). \tag{6}$$

The corresponding stress–energy tensor is

$$T_{\mu\nu} = \frac{\partial L}{\partial \phi_\nu}\phi_\mu - L\delta_{\mu\nu}. \tag{7}$$

The energy density is given by

$$T_{00} \equiv \mathcal{E}[\phi(x)] = \tfrac{1}{2}\phi_t^2 + \tfrac{1}{2}\phi_x^2 + U(\phi), \tag{8}$$

while the momentum density is

$$T_{10} \equiv \rho[\phi(x)] = \phi_t\phi_x. \tag{9}$$

We will calculate the force that is exchanged between two solitons: one placed in $x = x_1$ and the other in $x = x_2 = -x_1$. Then we will follow the motion of one of these solitons under the action of the interaction force.

For a solitary soliton we define the coordinate of the center of mass,

$$X_{\rm cm} = \frac{1}{E}\int_{-\infty}^{\infty} x\mathcal{E}[\phi(x)]\,\mathrm{d}x, \tag{10}$$

where $E = \int_{-\infty}^{\infty}\mathcal{E}[\phi(x)]\,\mathrm{d}x$. After some algebra with formulas (7)–(10), we conclude that the module of the static force exchanged between the two solitons is given by the expression

$$F = |T_{11}(x = 0, d)| = \tfrac{1}{2}\dot\phi^2(x = 0, d), \tag{11}$$

where $\phi(x, d)$ is the solution that describes the superposition of two solitons: one situated in point x_1 and the other in point x_2. Additionally, $d = x_2 - x_1$.

The solution for a static solitary soliton can be obtained from the equation

$$\tfrac{1}{2}\phi_x^2 = U(\phi). \tag{12}$$

Substituting the superposition of two soliton solutions from Eq. (12) in Eq. (11) (using (6)) for different n, yields the following asymptotic behavior,

$$F \sim \mathrm{e}^{-d}, \quad \text{for } n = 1, \tag{13}$$

while

$$F \sim d^{2n/(1-n)}, \quad \text{for } n > 1. \tag{14}$$

These results coincide with the ones obtained in Ref. [6] using other considerations.

Solitons with topological charge of equal sign interact with a repulsive force. When the topological charges are of opposite sign, the solitons attract each other.

However, formula (11) is very versatile and can be applied even when the distance between the solitons is small. In particular, it can be shown that the force between the solitons is finite for $d = 0$.

We can study numerically the dynamics of the interaction between two solitons. For this we will take Eq. (4) in an overdamped regime,

$$\phi_t = \phi_{xx} - 2\sin^{2n-1}\left(\tfrac{1}{2}\phi\right)\cos\left(\tfrac{1}{2}\phi\right), \tag{15}$$

with the aim to have the soliton velocity being proportional to the force that is acting on it. The initial condition corresponds to the superposition of two kinks situated at a finite distance,

$$\phi(x,0) = 4\left[\arctan\left(e^{x-x_0}\right) + \arctan\left(e^{x+x_0}\right)\right] - 2\pi, \tag{16}$$

$$\phi_t(x,0) = 0. \tag{17}$$

We wish to observe the motion of the soliton placed in the point $x_1 = x_0 > 0$. In this case the center of mass will be defined as

$$X_{\mathrm{cm}} = \frac{1}{E}\int_0^\infty x\mathcal{E}[\phi(x)]\,\mathrm{d}x, \tag{18}$$

where $E = \int_0^\infty \mathcal{E}[\phi(x)]\,\mathrm{d}x$. The soliton is repelled and its velocity will decay as it moves away.

When $n = 1$ the soliton moves away slightly and then it stops. This is because the interaction decays exponentially and when the solitons are separated at some distance they do not "feel" the interaction anymore.

For $n > 1$ the approximate decay law of the velocity coincides with the decay law of the force (14). Fig. 1 shows the log–log dependence of the soliton velocity on the distance for different n. The perfection of the power-law is striking.

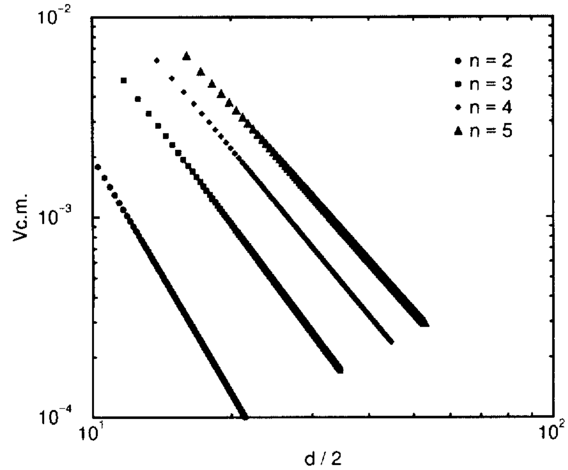

Fig. 1. Power-law dependence of the soliton velocity on the distance from the other interacting soliton. The slopes are 3.9 for $n = 2$, 3.0 for $n = 3$, 2.7 for $n = 4$ and 2.5 for $n = 5$.

3. A generalized Ginzburg–Landau equation

In this section we will study a generalization of the Ginzburg–Landau equation,

$$\frac{\partial u}{\partial t} = \nabla^2 u + u(1 - |u|^2)^{2n-1}. \tag{19}$$

Note that for $n = 1$ we recover the well-known Ginzburg–Landau (GL) equation [1,4,5].

Even when $n > 1$, Eq. (19) preserves all the topological properties of the original Ginzburg–Landau equation. There exists an unstable state at $u = 0$ and a degenerate stable state at $|u| = 1$. Alike the GL equation, Eq. (19) possesses topological solitons.

There is, however, an important difference. For $D = 1$ and $n = 1$, GL solitons interact with a force that decays exponentially with the distance. The soliton solutions of Eq. (19) ($n > 1$) interact with long-range forces as the long-range Klein–Gordon equation. Nevertheless, in this section, we will be interested in the case $D = 2$.

Using the transformations $U = \rho e^{i\theta}$ Eq. (19) can be rewritten as a pair of coupled differential equations,

$$\dot{\rho} = \nabla^2\rho - \rho|\nabla\theta|^2 + \rho(1-\rho^2)^{2n-1}, \tag{20}$$

$$\dot{\theta} = \nabla^2\theta + 2\rho^{-1}\nabla\rho\nabla\theta. \tag{21}$$

A vortex-like topological defect with topological charge K can be expressed in polar coordinates (r, ϕ) by the following equations,

$$\theta = K\phi, \tag{22}$$

$$\frac{\mathrm{d}^2R}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}R}{\mathrm{d}r} - \frac{K^2R}{r^2} + R(1-R^2)^{2n-1} = 0, \tag{23}$$

where we have defined $\rho \equiv R(r)$.

The configurations with topological charge $|K| = 1$ are stable.

The analysis of the asymptotic behavior of the vortex solution of Eq. (23) yields that in the limit $r \to \infty$

$$R(r) - 1 \sim r^{-2}, \tag{24}$$

when $n = 1$.

If $n > 1$, then

$$R(r) - 1 \sim r^{1/(1-n)}. \tag{25}$$

In the case of the generalized GL equation (19), the force that acts on a vortex situated in point r due to the existence of another vortex in the origin of coordinates satisfies the relation $F \sim R(r) - 1$. Thus, the vortices produced by Eq. (19) with $n = 1$ have Coulomb interaction [5]. Meanwhile, for $n > 1$ the interaction decays much more slowly.

4. Non-extensivity

The systems with long-range microscopic interactions can exhibit non-extensive behavior [19]. Recently, some alternative thermostatistics theories have been formulated [20].

In principle, it is possible to construct a system described by equations of the type

$$\frac{\partial^2\phi_1}{\partial t^2} + \gamma\frac{\partial\phi_1}{\partial t} - \nabla^2\phi_1 = -\frac{\partial V(|\phi_1|,|\phi_2|)}{\partial\phi_1}, \tag{26}$$

$$\frac{\partial^2\phi_2}{\partial t^2} + \gamma\frac{\partial\phi_2}{\partial t} - \nabla^2\phi_2 = -\frac{\partial V(|\phi_1|,|\phi_2|)}{\partial\phi_2}, \tag{27}$$

where potential $V(|\phi_1|,|\phi_2|)$ holds the necessary conditions in order to produce long-range interactions.

When we are in the presence of a system of equations like (26), (27) with two order parameters, we can have the situation where the sustained topological defects repel each other at very small distances and they attract each other at great distances [21].

We can have an effective interaction potential like the following,

$$V(r) = \mathcal{E}\left[\left(\frac{\sigma}{1+r^2}\right)^{\rho/2} - \left(\frac{\sigma}{1+r^2}\right)^{\alpha/2}\right], \tag{28}$$

where $\alpha < \rho$.

This is a situation equivalent to that discussed in Ref. [19]. Thus, when we have N particles in the system, the energy will grow with N following the laws

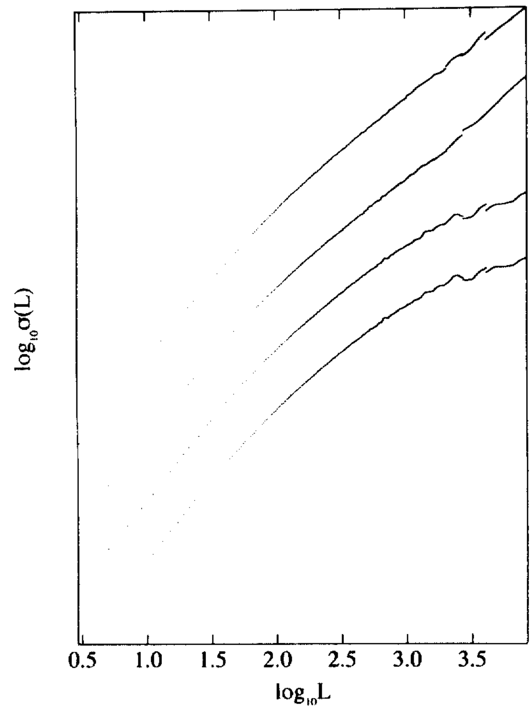

Fig. 2. Stationary regimes of the growth model described by Eq. (30) for $n = 1$ (lower curve), $n = 10$ and $n = 20$ (the two following curves), and $n = 40$ (upper curve). Note that for $n \gg 1$ self-affinity extends to greater scales.

$$\begin{aligned} E &\sim N && \text{if } \alpha/D > 1,\\ &\sim N\ln N && \text{if } \alpha/D = 1,\\ &\sim N^{2-\alpha/D} && \text{if } \alpha/D < 1. \end{aligned} \tag{29}$$

In our case $\alpha = (2-n)/(n-1)$. When $n > 1$ we are deeply in the non-extensive regime.

5. Growth models and pattern formation

Recently there has been a great interest in fractal surface growth [22,23]. Special attention has deserved the well-known KPZ equation [24]. Nonetheless, other equations have been studied as well [25,26], including the sine-Gordon model.

In this section we present an alternative model which is given by the equation

$$\phi_{tt} + \gamma\phi_t - \nabla^2\phi - G(\phi) = \eta(x,t), \tag{30}$$

where $\eta(x,t)$ is a white noise with the properties $\langle\eta(x,t)\rangle = 0$, $\langle\eta(x,t)\,\eta(x',t')\rangle = 2D\delta(t-t')\,\delta(x-x')$. Here the potential $U(\phi) = 2\sin^{2n}(\frac{1}{2}\phi)$, $G(\phi) = -\partial U(\phi)/\partial\phi$, holds the long-range interaction conditions discussed above.

This model presents noise-induced pattern formation. For $n = 1$ we recover the sine-Gordon equation. In this case, when the noise is small and the soliton–antisoliton pairs are not being created yet, the roughening exponent is zero. After the creation of the solitons we observe a crossover from a non-KPZ behavior ($\xi \sim 0.7$–0.8) to a KPZ behavior ($\xi \sim 0.5$). However, for great scales, there is a plateau with $\xi = 0$. The sine-Gordon equation does not eliminate the disorder at great scales.

Eq. (30) with $n > 1$ can be used as growth model for periodic media with marginal stability [8]. In this case, the activated solitons possess long-range interaction. Fig. 2 shows the stationary regimes for different n. For $n \gg 1$ self-affinity extends to all scales. Unlike the case $n = 1$, the surface in case $n = 40$ presents only two self-affine regimes, the anomalous ($\xi \sim 0.818$) and the KPZ-like ($\xi \sim 0.5$). The system displays fractal behavior at all scales.

We are interested in the process of pattern formation in Eq. (30). For this purpose we apply the wavelet transform analysis [27]. Fig. 3 shows the wavelet expansion of the stationary state for $n = 40$, $\gamma = 0.252$. We have used the "Mexican hat" wavelets, where a and b are the scale and position parameter, respectively. The existence of structures at different scales is evident. There is also local self-similarity. The pitchfork patterns are linked with fractal order.

6. Long-range interacting solitons in nature

In addition to the physical phenomena already mentioned throughout the paper, in this section we briefly discuss some other physical systems where there is evidence for the long-range interactions between the topological solitons appearing there.

It is well-known that topological solitons play a very important role in nonlinear quantum field theories [28–36]. Extended-particle states in quantum field theories can be described by solitons based on classical solutions to the field equations. The corresponding particle-like solutions are equivalent to fermions.

The main postulate in these theories is a Lagrangian density,

$$L = \tfrac{1}{2}\phi_t^2 - \tfrac{1}{2}\phi_x^2 - U(\phi), \tag{31}$$

where $U(\phi)$ possesses (at least) two degenerate minima, corresponding to the vacuum solutions. This model describes a self-interacting scalar field. In some cases the Lagrangian can include the interaction of field ϕ with other quantum (vector, spinor) fields.

The most common models use the sine-Gordon and ϕ^4 equations [28–33]. In this case, the topological solitons can be interpreted as hadrons [28–33] (strongly interacting particles). For these particles, interaction has a short-range character. Nevertheless in nature, there exist also elementary particles with long-range interactions [6,31].

Note that when the potential $U(\phi)$ behaves as $U(\phi) \sim (\phi - \phi_i)^{2n}$ ($n > 1$) near the vacuum values, we obtain

$$\left[\frac{\partial U(\phi)}{\partial\phi}\right]_{\phi=\phi_i} = 0. \tag{32}$$

But this is equivalent to saying that the field is massless [6,8,31]. A well-established fact in elementary particle physics is that a massless field leads to the existence of long-range interacting particles [31]. However, if we put $m = 0$, for instance, in the ϕ^4-theory Lagrangian [29]

$$L = \tfrac{1}{2}\phi_t^2 - \tfrac{1}{2}\phi_x^2 + \tfrac{1}{2}m^2\phi^2 - \tfrac{1}{4}\lambda\phi^4 - \tfrac{1}{4}m^4/\lambda, \tag{33}$$

we do not have solitons anymore. For this we need a $U(\phi)$ with two or more minima [6].

So, if we wish to describe these particles using soliton solutions in a theory like (31) (and not with a linear theory as QED does), in order to derive the properties of the particles and their interaction from the Lagrangian, then we should require a massless field and a multi-degenerate vacuum. And these properties are contained in the nonlinear Klein–Gordon model discussed in this Letter.

In Ref. [37] the authors study the phase-locking in coupled long Josephson junctions. In Josephson junctions the fluxons are described by topological solitons of the sine-Gordon equation. However, if two or more long Josephson junctions are coupled, the external magnetic fields created outside the junctions will

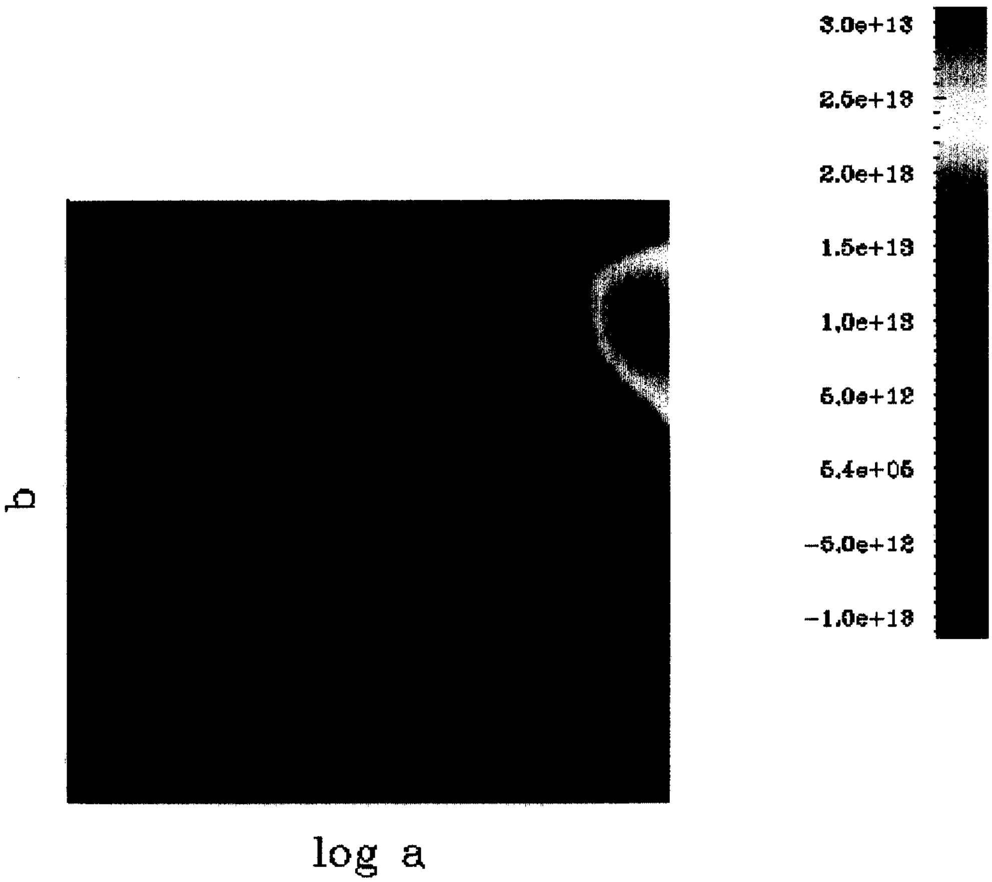

Fig. 3. Wavelet expansion of the $\phi(x,t)$ profile produced by Eq. (30). This figure shows the existence of coherent structures at different scales.

overlap producing long-range interactions. The experimental results presented in several papers [38–41] show that the magnetic interaction is strong enough to produce substantial phase-locking between the fluxons.

The assumption in Ref. [37] is that the external magnetic coupling leads to a nonlocal interaction. In fact, there is experimental evidence that can be interpreted as a manifestation of the long-range interaction between the fluxons.

An array of two long Josephson junctions can be modeled by a system of two coupled modified sine-Gordon equations which are a particular case of the set of equations (26), (27). Thus, the results obtained in the present Letter concerning the long-range interaction between topological solitons could be applied to the fluxons in magnetically coupled long Josephson junctions.

In the Yakushevich model for solitons in the DNA torsional dynamics [7,43], the equations of motion are

$$I_1\phi_{1tt} - K_1 a^2 \phi_{1zz} = -\frac{\partial U}{\partial \phi_1}, \tag{34}$$

$$I_2\phi_{2tt} - K_2 a^2 \phi_{2zz} = -\frac{\partial U}{\partial \phi_2}, \tag{35}$$

where

$$U(\phi_1,\phi_2) = \tfrac{1}{2}kR^2\left\{\left[\left(2+\frac{l_0}{R}-\cos\phi_1-\cos\phi_2\right)^2 + (\sin\phi_1-\sin\phi_2)^2\right]^{1/2} - \frac{l_0}{R}\right\}^2, \tag{36}$$

For an explanation of the variables, parameters and a theoretical investigation see Refs. [7,43].

It can be shown [7,21] that the potential $U(\phi_1, \phi_2)$ possesses the degeneracy properties discussed in the present Letter. Therefore the solitons can interact with long-range forces which could explain some of the long-range effects observed in this molecule [15–17].

Another important phenomenon in biophysics is the formation of protein structure [44] (protein folding). Recent studies [45] consider that this process is controlled by long-range excitations dynamically induced along the backbones of protein molecules. We think that the results obtained here could help in formulating a new model of protein folding and/or the interpretation of experimental data.

7. Conclusions

The nonlinear Klein–Gordon equation (1) with a potential $U(\phi)$ that satisfies the degeneracy properties discussed in this Letter (see formula (2)) possesses solitonic solutions that interact with long-range forces. In particular, we investigated the model described by Eq. (4). For $n = 1$ (the sine-Gordon equation), the interaction force decays exponentially, whereas for $n > 1$ the interaction force decays with the distance as $F \sim d^{2n/(1-n)}$. The numerical experiments with Eq. (15) confirm this result.

The Ginzburg–Landau equation can be generalized in such a way that the topological defects supported by this equations present long-range interaction both in $D = 1$ and $D > 1$.

In particular, when $D = 2$ we have the following behavior: $F(d) \sim d^{-2}$ for $n = 1$. On the other hand, $F(d) \sim d^{1/(1-n)}$ for $n > 1$.

A system of two equations with two complex order parameters can be constructed in such a way that the interaction force between the topological objects decays so slowly that we enter the non-extensivity regime. This is important in order to study a whole series of physical systems from vortices in liquid helium to cosmic strings [46].

Systems of interacting particles with long-range interactions exhibit a very complex dynamics that cannot be described by the Boltzmann–Gibbs statistics. Recently, alternative thermostatistical theories have been formulated. The models studied in this Letter could help to verify these theories.

The growth model described by Eq. (30) has interesting features. It experiences a transition to an ordered state associated with the activation of a soliton gas. There is a crossover from an anomalous non-KPZ behavior to a KPZ behavior. However, unlike the sine-Gordon equation, in our model the self-affinity extends to all scales.

Note that it is the KPZ-like behavior that extends to infinity. This is because the KPZ regime is related to the absence of a mass term in the evolution equation [8]. And this is the case of Eq. (4) for $n > 1$. The anomalous regime is due to the existence of soliton solutions. The solitons are present both in the sine-Gordon equation and the long-range Klein–Gordon equation. On the other hand, the KPZ equation is not a soliton-bearing system. These considerations explain the existence of two regimes in the long-range Klein–Gordon equation, unlike the KPZ equation and the sine-Gordon equation.

The wavelet analysis permitted to observe the existence of coherent structures at all scales.

In general, the models we have studied can describe a "world" where there is spontaneous formation of topological objects with long-range interactions, which can create complex structures showing fractal behavior.

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